Supplemental Document

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Single-input polarization-sensitive optical coherence tomography through a catheter: supplement

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Supplementary Material

S.1: Optimization of Symmetric Compensation Estimation

When processing PS-OCT data with spectral binning, after making cumulative matrices transpose symmetric, we 'align' the matrices of the various spectral bins to the central spectral bin. Operating in SO3, we are looking for

$$\min_{\mathbf{Q}} \sum_{n} \left\| \mathbf{D} \cdot \mathbf{Q} \cdot \mathbf{D} \cdot \mathbf{G}_{\mathrm{n}} \cdot \mathbf{Q}^{\mathrm{T}} - \mathbf{W}_{\mathrm{n}} \right\|_2^2$$

where all matrices are normalized SO3 rotation matrices, and we use the fact that $\mathbf{Q}^{-1} = \mathbf{Q}^{\mathbf{T}}$. $\mathbf{W}$ is the rotation matrix of the central bin and $\mathbf{G}$ that of any other bin. $\mathbf{D}$ = diag([1,1,-1]). Both $\mathbf{W}$ and $\mathbf{G}$ are D-transpose symmetric. The sum is taken over all pixels n available (with sufficiently low depolarization). $\mathbf{Q}$ is 'aligning' the cumulative rotation matrix of $\mathbf{G}$ with that of the central bin.

Using the Kronecker product rule, we can rewrite this minimization problem as:

$$\min_{\mathbf{Q}} \sum_{n} \left\| (\mathbf{Q} \otimes \mathbf{D} \cdot \mathbf{Q} \cdot \mathbf{D}) \cdot \overrightarrow{\mathbf{G}_{\mathrm{n}}} - \overrightarrow{\mathbf{W}_{\mathrm{n}}} \right\|_2^2 = \min_{\mathbf{Q}} \sum_{n} \left\| \mathbf{X} \cdot \overrightarrow{\mathbf{G}_{\mathrm{n}}} - \overrightarrow{\mathbf{W}_{\mathrm{n}}} \right\|_2^2$$

The corresponding metric can be expressed as

$$\varepsilon = \sum_{n} \mathrm{tr}\left(\overrightarrow{\mathbf{G}_{\mathrm{n}}}^{T} \cdot \mathbf{X}^{\mathrm{T}} \cdot \mathbf{X} \cdot \overrightarrow{\mathbf{G}_{\mathrm{n}}}\right) + \sum_{n} \mathrm{tr}\left(\overrightarrow{\mathbf{W}_{\mathrm{n}}}^{T} \cdot \overrightarrow{\mathbf{W}_{\mathrm{n}}}\right) - \sum_{n} \mathrm{tr}\left(\overrightarrow{\mathbf{W}_{\mathrm{n}}}^{T} \cdot \mathbf{X} \cdot \overrightarrow{\mathbf{G}_{\mathrm{n}}}\right) - \sum_{n} \mathrm{tr}\left(\overrightarrow{\mathbf{G}_{\mathrm{n}}}^{T} \cdot \mathbf{X}^{\mathrm{T}} \cdot \overrightarrow{\mathbf{W}_{\mathrm{n}}}\right)$$

$$\varepsilon = \sum_{n} \mathrm{tr}\left(\overrightarrow{\mathbf{G}_{\mathrm{n}}}^{T} \cdot \overrightarrow{\mathbf{G}_{\mathrm{n}}}\right) + \sum_{n} \mathrm{tr}\left(\overrightarrow{\mathbf{W}_{\mathrm{n}}}^{T} \cdot \overrightarrow{\mathbf{W}_{\mathrm{n}}}\right) - 2 \sum_{n} \mathrm{tr}\left(\overrightarrow{\mathbf{G}_{\mathrm{n}}} \cdot \overrightarrow{\mathbf{W}_{\mathrm{n}}}^{T} \cdot \mathbf{X}\right)$$

$$\varepsilon = \sum_{n} \mathrm{tr}\left(\overrightarrow{\mathbf{G}_{\mathrm{n}}}^{T} \cdot \overrightarrow{\mathbf{G}_{\mathrm{n}}}\right) + \sum_{n} \mathrm{tr}\left(\overrightarrow{\mathbf{W}_{\mathrm{n}}}^{T} \cdot \overrightarrow{\mathbf{W}_{\mathrm{n}}}\right) - 2 \sum_{n} \mathrm{tr}\left(\mathbf{X}^{\mathrm{T}} \cdot \overrightarrow{\mathbf{W}_{\mathrm{n}}} \cdot \overrightarrow{\mathbf{G}_{\mathrm{n}}}^{T}\right),$$

because by construction $\mathbf{X}$ is unitary.

$$\begin{aligned}
&(\mathbf{A} \otimes \mathbf{B})^{T} \cdot (\mathbf{A} \otimes \mathbf{B}) \\
&= \left(\mathbf{A}^{\mathrm{T}} \otimes \mathbf{B}^{\mathrm{T}}\right) \cdot (\mathbf{A} \otimes \mathbf{B}) \\
&= \left(\mathbf{A}^{\mathrm{T}} \cdot \mathbf{A}\right) \otimes \left(\mathbf{B}^{\mathrm{T}} \otimes \mathbf{B}\right) \\
&= \mathbf{I}.
\end{aligned}$$

Hence, minimizing epsilon is identical to maximizing the rightmost term, which can be further written as

$$2\,\mathrm{tr}\left(\mathbf{X}^{\mathrm{T}}\cdot\sum_{n}\left(\overrightarrow{\mathbf{W}_{\mathrm{n}}}\cdot\overrightarrow{\mathbf{G}_{\mathrm{n}}}^{T}\right)\right).$$

Hence, we can simply collect the summed term, and then optimize **Q** using iterative optimization, which is computationally very cheap, since we are multiplying it with a single 9x9 matrix.

S.2: Singular Value Decomposition Rationale

The SIPS formalism relies on finding the maximization of the following expression

$$\begin{aligned}\tilde{\mathbf{j}} &= \arg\max_{\mathbf{j}} \mathbf{j}^{\dagger}\cdot\mathbf{P}^{\dagger}\cdot 2\mathbf{M}\cdot\mathbf{P}\cdot\mathbf{j}\\ &= \arg\max_{\mathbf{j}} \mathbf{j}^{\dagger}\cdot\mathbf{H}\cdot\mathbf{j}.\end{aligned}$$

In the following derivation, we will show that this is equivalent to the singular value decomposition of the real part of the Hermitian matrix **H**.

The gradient of this expression with respect to **j**, where we restrict **j** to be real, is

$$\nabla\,\mathbf{j} = \frac{\partial\left(\mathbf{j}^{\dagger}\cdot\mathbf{H}\cdot\mathbf{j}\right)}{\partial\mathbf{j}} = \mathbf{H}\cdot\mathbf{j} + \mathbf{H}^{\mathrm{T}}\cdot\mathbf{j} = 2\Re\{\mathbf{H}\}\cdot\mathbf{j}.$$

Taking the derivative of the norm of **j**, which must be conserved, we find that

$$\nabla\,\mathbf{j}^{\mathrm{T}}\cdot\mathbf{j} = 0,$$

i.e., the derivative of **j** is confined to the tangent space on the unit sphere at **j**. Hence, we have to project the gradient of the full expression into a direction orthogonal to **j**, which is done by

$$\nabla\,\mathbf{j}' = \nabla\,\mathbf{j} - \left(\mathbf{j}^{\mathrm{T}}\cdot\nabla\,\mathbf{j}\right)\mathbf{j}$$

Now setting this projected gradient to zero gives

$$\nabla\,\mathbf{j} = \left(\mathbf{j}^{\mathrm{T}}\cdot\nabla\,\mathbf{j}\right)\mathbf{j}$$

$$\Re\{\mathbf{H}\}\cdot\mathbf{j} = \left(\mathbf{j}^{\mathrm{T}}\cdot\Re\{\mathbf{H}\}\cdot\mathbf{j}\right)\mathbf{j}$$

which is fulfilled for any of the singular vectors of $\mathrm{Re}\{\mathbf{H}\}$. Picking the largest one gives the optimal solution.